MATH 1401 SPRING 2000 CHEAT SHEET FINAL

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1. Important formulas from algebra. $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$, $\sin^2 x + \cos^2 x = 1$, $a^{b+c} = a^b a^c$, $a^{m/n} = \sqrt[n]{a^m}$, $a^b = e^{(\log a)b}$. Solution of $ax^2 + bx + c = 0$ is $x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

2. Limits and continuity. $\lim_{x\to c} f(x) = f(c) \iff f$ is continuous at c
$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$, $\lim_{x\to 0} \frac{1-cos(x)}{x} = 0, \lim_{x\to 0}(1+x)^{1/x} = e$
$\lim_{x\to c} f(x) = L \iff \lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = L$
Intermediate value theorem: If f is continuous on [a,b] and k is between $f(a)$ and $f(b)$, then there exists $c \in [a,b]$ such that $f(c) = k$.
Infinite limits: The formulas for the limit of sum, product, and quotient apply unless they lead to undefined expressions of the form $\infty - \infty$, $\infty.0$, $L/0$, ∞/∞.
If $\lim_{x\to c} f(x) \neq 0$ and $\lim_{x\to c} g(x) = 0$, with $g(x) \neq 0$ on a neighborhood of c, then the graph of f/g has vertical asymptote $x = c$.

3. Differentiation. The equation of the line passing through (x_0, y_0) with slope s is $y - y_0 = s(x - x_0)$. The equation of the tangent to the graph of f at (x_0, y_0), $y_0 = f(x_0)$, is $y - y_0 = f'(x_0)(x - x_0)$.
$f'(c) = \lim_{x\to c}(f(x) - f(c))/(x - c)$. If $f'(c)$ exists, f is continuous at c.
$(x^n)' = nx^{n-1}$, $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(\ln x)' = 1/x$, $(e^x)' = e^x$
$\sin' x = \cos x$, $\cos' x = -\sin x$, $(\arctan x)' = 1/(1 + x^2)$, $\arcsin' x = \frac{1}{\sqrt{1-x^2}}$, $\text{arcsec}' x = \frac{1}{|x|\sqrt{1-x^2}}$, $(uv)' = u'v + uv'$, $(u/v)' = (u'v - uv')/v^2$, $f(g(x)) = f'(g(x))g'(x)$
If $g = f^{-1}$ and $y = g(x)$, $f'(y) \neq 0$, then $g'(x) = 1/f'(y)$.

4. Applications and extrema. If f is continuous on $[a,b]$, then f attains maximum and minimum on $[a,b]$. f can attain extremum on $[a,b]$ only at endpoints or critical numbers (where f' does not exist or $f' = 0$). f can attain relative extremum in (a,b) only at a critical number.
Mean value theorem: If f is continuous on $[a,b]$ and differentiable on (a,b) then there exists $c \in (a,b)$ such that $f'(c) = (f(b) - f(a))/(b - a)$. (The case when $f(a) = f(b)$ is Rolle's theorem.)
If $f' > 0$ in (a,b) and f is continuous on $[a,b]$, then f is increasing on $[a,b]$.
If f is continuous at c, $f'(x) < 0$ for $x < c$ and $f'(x) > 0$ for $x > c$, then f has relative minimum $(c, f(c))$. (Or, relative minimum $f(c)$ at $x = c$.)
If f' in increasing in interval I, then f is concave upward in I.
If $f'' > 0$ in (a,b), then f is concave upward in (a,b).
If $f'(c) = 0$ and $f''(c) > 0$, then f has relative minimum at c.

5. Hyperbolic functions. $\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$, $\cosh^2 x - \sinh^2 x = 1$, $\cosh' x = \sinh x$, $(\tanh^{-1})' = 1/(1 - x^2)$

6. Integration. $\int f(x)\,dx = F(x) + C$, $F' = f$.
$\int x^n\,dx = x^{n+1}/(n+1) + C, n \neq -1$, $\int f(g(x))g'(x)\,dx = \int f(u)\,du$, $u = g(x)$
$\int \frac{1}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$, $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int \frac{1}{\sqrt{a^2+x^2}} = \sinh^{-1} \frac{x}{a} + C$, $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{a} \tanh^{-1} \frac{x}{a} + C$
$\int_a^b f(x)\,dx = F(b) - F(a)$, $F' = f$.
$(d/dx) \int_a^x f(t)\,dt = f(x)$